Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

XVII.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF HARVARD UNIVERSITY.

A PAPER ON THE PROPAGATION OF MAGNETIC WAVES IN SOFT IRON.

By Harold Whiting.

Presented by Prof. Trowbridge, Feb 14, 1881.

When a rapidly alternating current is made to pass through a coil of insulated wire wound about one end of a bar of soft iron, the latter becomes the seat of a series of magnetic waves, whose velocity and manner of propagation have, I think, not been determined.

The object of the present paper is to show that these waves differ in form, phase, and magnitude from the values assigned to them by any possible theory of instantaneous propagation; and that this rate of propagation, as determined by actual observation, is not generally very great, being measured in ordinary experiments by only a few feet, or at most a few hundred feet per second: but that no fixed velocity of propagation can be said to exist; since it depends largely upon the period of the wave, the material, dimensions, and accidental conditions of the bar, the distance from the magnetizing coil, and even, in extreme cases, upon the strength and distribution of the magnetic field.

In developing the complex laws which determine the velocity of propagation under so many variable conditions, I shall try also to show that the close analogy between magnetism, electricity, and heat, may be traced throughout the phenomena of magnetism in motion.

As early as the spring of 1876 I had completed, under the direction of Professor Trowbridge, a series of measurements supplementary to those of Mr. Rowland, showing the distribution of magnetism over a soft iron bar applied at one end to the pole of a permanent magnet. The subject next assigned to me was naturally the rate of this distribution, and of the propagation of magnetism in general.

A long half-inch bar was magnetized at one end by a (primary) coil connected with a battery. A small sliding secondary, of about 15 turns, was clamped at a short distance along the bar, and somewhat further off was placed a larger one, of about 150 turns.

The two coils were connected (in series) with a galvanometer so as to give opposed induction effects, and their relative distance was varied so that these effects should neutralize each other as nearly as possible.

It was found, as expected, that when the latter adjustment was made as accurately as possible, the spot of light thrown by the mirror of the galvanometer still made invariably a sudden motion, as if under the influence of the nearer of the two coils, followed by a sudden stop, due supposably to the somewhat tardier action of the more distant.

The interval of time between the two effects was estimated by various methods sufficiently obvious; the most successful being the actual determination of how rapidly the circuit must be closed and broken in order to reproduce, with a single secondary, the same effects as before. The various distances between the two secondaries, divided by the corresponding intervals of time, gave, of course, certain values of the velocity of propagation. These all lay within a few feet of 30 per second, for distances of not over one foot, beginning at 2 inches from the primary. How nearly these earlier estimates agree with the results of later investigation may be found below.

The method of measurement, however, is open to serious criticism, since the first sudden motion of the spot of light might be due to the small counter-current which ordinarily precedes the arrival of the normal current, and the magnitude of the counter-current would doubtless be greater in the larger of the two coils; so that the first sudden motion might really be due to the action of the more distant, rather than to that of the nearer coil. The experiments were, therefore, discontinued.

It was not until the autumn of 1880 that the experiments were again resumed, no new method less open to criticism having suggested itself in the mean time. I should, however, mention that certain experiments of Professor Trowbridge at Newport (which will doubtless be published in due time) had added both a confirmation of previous results and a renewed interest in the continuation of the investigation.

For the sake of continuity, the earlier experiments were repeated; then varied by the substitution of one secondary and two primaries

for two secondaries and one primary. Though the new method of procedure was open to the same objections as the old, the figures all agreed, within sufficiently narrow limits, both with themselves and with those before obtained. A telephone and musical contact-breaker were now substituted for the galvanometer and key. It was expected that the two sets of waves induced by a single primary in two secondaries, or by two primaries in a single secondary, could be made to interfere.

It was, of course, possible (upon any theory) so to place the two coils that each should neutralize very nearly the effect of the other, the telephone then being silent, and any considerable deviation from this position of the two coils must necessarily be accompanied by the production of a musical note; but, after long and unsuccessful search for *other* maxima and minima, the inevitable conclusion was reached that no true phenomenon of magnetic wave-interference could be found, and it was feared that all the previous results might have been delusive.

It was afterward discovered that, although the wave suffers a retardation, this retardation diminishes with the wave-length, so that it cannot produce complete interference under any condition whatsoever. We shall see, in fact, that the phenomena which have been described are perfectly consistent, the methods sound, and even that the rough measurements of these earlier experiments may be considered as close approximations to the truth.

The apparatus, principally employed in later and more precise determinations (see Diagram), consisted of a wooden shaft, one fourth inch in diameter, and about one foot long, carrying a pair of commutators (cut from a convenient size of brass tubing), — one of which was of the ordinary sort, the other different from it, in that its two arms were greatly prolonged, and at the same time twisted, corkscrew fashion, round the shaft one and one half turns each. The two lines of separation formed a pair of spirals of nearly constant pitch, so that by means of sliding contact-pieces this commutator might be made to act either simultaneously with its neighbor, or any fraction of a revolution sooner or later,* and this without stopping the machine. One of the commutators was put in the primary, the other in the secondary, circuit. The primary, or magnetizing coil, was of No. 16 insulated wire, wound to a depth of half an inch over the first three inches of the

* This fraction of a revolution is called, later on, the inclination of, or angle between, the two commutators.

bar. The secondary was of No. 36 wire, wound upon a bobbin, with half an inch between its flanges, to a depth of about three eighths of an inch, and large enough to slide freely over the half-inch iron rod used in most of the experiments. A galvanometer, of somewhat greater resistance than the secondary coil, was also included in its circuit. The common shaft of the two commutators carried a fly-wheel and revolution-counter, and was set in motion by a pulley-wheel connected by a band with a powerful electric motor.

Though different in detail from the "Differential Interruptor" of Professor Blaserna (as described in Gordon's "Electricity and Magnetism," vol. i. p. 311 and ff.), this machine, though independently constructed, bears a most striking similarity to that of the Italian professor. The latter needs only the introduction of an iron core, common to the two secondaries, to fit it for the determinations which follow. The former is, however, adapted to the use of an alternating, instead of an intermittent, current, which gives for magnetic measurements a special advantage, and in determining not the slope of a given portion of the primary wave, but the value of the whole area enclosed by it, would consistently have been called by Professor Blaserna an "Integral Alternator." It being, however, the only *machine* employed in the phase-determinations which follow, I have alluded to it, when necessary, by its generic name.

When the shaft is set in rapid rotation, the bar is evidently magnetized; first positively, then negatively, &c. The period of time elapsing between two successive maxima of the same sign, in no matter what portion of the bar, evidently agrees with the length of time occupied by a single revolution of the commutator, and may be called the period of the magnetic wave. Each period repeats, of course, the same continuous series of changes as the one before it. The wave, accordingly, possesses a true *phase;* and the difference of phase of the magnetic wave at two different points in the bar may be defined as equal to the arc through which the shaft of the commutator must revolve, after the wave has reached a certain phase of its development in the one locality, in order that the tardier portion of the same wave in the other locality may attain the same, or rather a corresponding phase of *its* development. It is easy to find four corresponding points in the phase of any two waves; namely, the two maxima, positive and negative, and the two half-way points, or points of zero displacement. By means of one of the latter points the differences of phase were in all cases determined.

The current induced in the secondary coil would naturally flow one

way, while the magnetic wave is rising within it, and the other way, while falling; giving no effect, as a whole, upon the galvanometer. The second (adjustable) commutator, already alluded to, made it possible to "redress"* these currents, so that they should *all* pass through the galvanometer in *either* direction. In fact, it was almost impossible so to adjust it that the current should *not* be turned, on the whole, a little one way or the other.

To avoid jarring, and direct action of the magnetism upon the galvanometer, whose deflections were to be noted, the machinery was removed as far as possible from the observer; while an assistant, according to direction, varied the angle between the two commutators, by means of the sliding-piece already alluded to, until the average deflection of the galvanometer was reduced to zero. Now the effect of the commutator in the secondary circuit is to add together all the pulses, positive and negative, which pass through it, in either of its two positions, to the galvanometer; and the symmetry of the wave, as regards positive and negative displacements, shows that the pulses are all alike; hence, each must be equal to zero when the total effect is zero. Since the magnitude of the pulse for a given interval of time is proportional to the rise or fall of the wave during that time, this rise or fall must also be equal to zero; that is, the initial and final displacements are identical: and hence (again from principles of symmetry) it is evident that, if there be no deflection of the galvanometer, every magnetic wave must have reached its mean height within the secondary coil at the instant of action of the (adjustable) commutator included in the circuit; so that comparison of these angles of adjustment for different velocities of the machinery, different distances between the two coils, and for different bars served to determine the various rates of propagation of the magnetic wave under these different conditions.

Earlier experiments gave discordant results, owing partly to imperfect contact within the commutator (which was afterward remedied as far as possible), but especially on account of the change of "magnetic conductivity" (if I may use the term) which the bar appeared to undergo after a few minutes of rapid and powerful magnetization and demagnetization. In one case, the conductivity appeared in consequence to increase two or three fold.

I should have been at some pains still further to investigate (or at least to confirm) this sensitiveness of soft iron to the influence of a

* In the sense of the French word, *redresser*.

strong alternating current, had I not been informed by Professor Trowbridge that experiments at Newport had already placed the matter beyond all doubt.

It had been supposed that only a powerful battery (ten or more Bunsen cells) would be able to give distinctly measurable results for the remote portions of the bar, whereas it now appeared that the conditions of success in these experiments depend upon the insufficiency of each current to produce a condition of magnetic saturation within the limited time of its action. The use of a single cell obviated all these difficulties, and was found to give sufficiently precise results for all portions of the bar.

In order to determine the several positions of the sliding-piece which corresponded to the simultaneous action of the two commutators, the shaft was set revolving very slowly, and the contact-piece was moved until the spot of light thrown by the galvanometer, though feeling every pulse, showed no constant deflection to right or left. A fixed pointer then covered, in each case, a certain division of a scale attached to the sliding-piece, the number of which being noted served as a zero for the calculation of angular measurements. The branches of the commutator making not quite one and one half turns, there were at most three points which satisfied this condition, one of which was taken as 0°, the next as 180°, and the third as 360°. Intermediate inclinations of the two commutators were measured by simple interpolation. Owing to the wearing away of the centres which supported the shaft, and their occasional readjustment, these three points had frequently to be redetermined.

The result of a long series of experiments * upon a half-inch rod, some 53 inches long, was to show that the phase, in the portions of the bar not very remote from the primary, was *invariably later* than within the primary itself; and that this retardation increased up to a certain distance (not under 10 nor over 20 inches), to a maximum; and then diminished again, so that the furthest end of the bar did not differ in phase from the primary.

The phase of the primary was itself retarded more and more as the speed of the commutator increased, until for the highest velocities (100 or more reversals per second) it became fully equal to 90°, while the maximum retardation found anywhere in the bar never exceeded 127° or 130°; and was, accordingly, never more than 37° or 40° later than the primary.

* See Table for Phase Retardation, appended.

The apparent velocity of propagation over the first seven inches of the bar ranged from 25 feet per second, for the low velocity of four reversals per second, up to 90 feet for about 23 reversals; after which it rose nearly in proportion to the number of reversals, until for 140 reversals it measured about 300 feet per second. At distances greater than 7 inches, the direct and instantaneous action of the primary and adjacent parts of the bar predominates more and more over the pulse propagated through the medium of the iron; so that the investigation of the rate of propagation in distant portions of the bar is not instructive. Indeed, since the most remote portions of the bar agree in phase with the primary, while intermediate portions lag behind it, we shall find the magnetic wave apparently flowing backward in the most distant portions. This, however paradoxical it may seem, is strictly in accordance with both theory and fact.

The following set of observations was dated Jan. 11: —

$c =$ the arbitrary position of the coil.

$ph =$ the position of the sliding-piece which gave zero deflection of the galvanometer.

$V =$ the number of ticks of a watch (beating 288 times to a minute) made while the shaft was performing 72 complete revolutions, and counted before and after each set of observations.

$V = 84$ - - - 83.	
$c = 0$	$ph = 14\frac{1}{8}$ mm.
1	$14\frac{7}{8}$ „
2	$15\frac{1}{16}$ „
3	$15\frac{1}{4}$ „
4	$15\frac{3}{8}$ „
5	$15\frac{7}{8}$ „
7	$16\frac{1}{4}$ „
10	$16\frac{3}{4}$ „
15	$17\frac{1}{4}$ „
20	$17\frac{3}{8}$ „
30	$16\frac{1}{2}$ „
45	...
10	$16\frac{3}{4}$ „
0	$14\frac{1}{4}$ „

$V = 25$ - - - 29.	
$c = 0$	$ph = 18\frac{1}{4}$ mm.
2	$19\frac{1}{2}$ „
5	$20\frac{5}{8}$ „
10	$21\frac{1}{8}$ „
15	$22\frac{3}{8}$ „
20	24 „
25	24 „
30	$23\frac{1}{2}$ „
35	$22\frac{1}{2}$ „
40	$21\frac{1}{4}$ „
45	$20\frac{5}{8}$ „
48	$19\frac{3}{4}$ „

The first point of simultaneous action was $10\frac{1}{2}$ mm., the second $29\frac{1}{2}$ mm., and the third $38\frac{1}{2}$ mm. The distances were reckoned in inches, beginning at a point 2 inches from the centre of the primary.

Hence we get the following tables, where $v =$ the number of revolutions per second, $t =$ the length of time occupied by the wave in reaching the distance, d, measured in inches from the centre of the primary: —

$v = 4.$	
$t = .023$	$d = 2$
3	.028
4	.029
5	.029
6	.031
7	.034
9	.037
12	.037
17	.042
.22	.044
32	.038
47	...
12	.037
2	.024

$v = 14$ . . . 12.	
$d = 2$	$t = .015$
4	.017
7	.020
12	.021
17	.024
22	.027
27	:028
32	.027
37	.025
42	.023
47	.022
50	.020

The following observations, taken with a break corresponding to the removal of the first 10 inches of the bar, bear also the same date: —

$V = 26.$	
$c = 10$	$ph = 15\frac{5}{8}$ mm.
15	19 „
20	$18\frac{7}{8}$ „
30	$19\frac{1}{8}$ „
48	19 „

$v = 13.$	
$d = 12$	$t = .010$
17	.016
22	.016
32	.016
50	.016

showing that the interruption of a large portion of the pulse of propagation is accompanied by a much less retardation of the phase. There was reason to believe that only about half of this pulse was interrupted in this way. All attempts to measure the amount of the *direct* pulse have given widely different results ; and the rough experiments, upon which the above opinion was founded, do not deserve special notice.

The phenomena are essentially the same for a short bar of about 20 inches in length; that is, of course, as far as they go. In the notation used above, the results are reduced as follows: —

	$v = 4.5$	$v = 14.5$
$d = 2$	$t = .018$	$t = .012$
3	.021	.013
4	.022	.015
5	.023	.016
6	.024	.016
7	.025	.018
9	.028	.018
12	.033	.020
17	.034	.021

A small rod of one fourth inch diameter was now substituted, and much smaller differences of phase were obtained, corresponding to the

same speed of the commutator. A steel bar, one half inch in diameter, gave nearly the same results as the one fourth inch rod of soft iron, but the highest velocities were not used with any of them. It was, however, determined for both that an increase of electromotive force causes a considerable increase in the retardation, but apparently affects the soft iron more in the nearer, and the steel in the further, portions.

Little reliance can be placed in determinations for distances above 20 inches; but it would seem probable, from the experiments below, that the anomalous behavior of the steel is due to "coercive force," which in the most distant parts gives way only under the influence of a powerful battery, while that of the soft iron is due to supersaturation, which can exist only in the nearer portions.

It appears, also, from the experiments, that the "magnetic conductivity" of the steel cannot be far from a fourth of that of the soft iron.*

TABLE FOR HALF-INCH STEEL ROD.

$V = 81 - - - 86.$

	(1 Cell.)	(10 Cells.)
$c = 0$	$ph = 12.7$ mm.	$ph = 12.6$ mm.
1	13.0 „	13.1 „
2	13.2 „	13.4 „
3	13.2 „	13.5 „
4	13.2 „	13.6 „
5	13.2 „	13.6 „
7	13.2 „	13.6 „
10	13.4 „	13.6 „
15	13.6 „	13.6 „
20	13.6 „	13.6 „
30	13.? „	13.? „

$V = 20 - - - 22.$

	(10 Cells.)	(1 Cell.)
$c = 0$	$ph = 15.6$ mm.	$ph = 16.0$ mm.
1	16.2 „	16.6 „
2	16.7 „	16.9 „
3	16.9 „	17.4 „
4	17.1 „	17.5 „
5	17.6 „	17.8 „
7	18.0 „	18.2 „
10	18.4 „	18.7 „
15	19.0 „	18.5 „
20	19.5 „	17.7 „
25	21.0 „	...
30	24.? „	14.5? „

* See "Rowland's Tables of Permeability of Steel and Iron," American Journal of Science, 1873.

TABLE FOR QUARTER-INCH IRON ROD.

$V = 24 - - - 27.$

	(1 Cell.)	(4 Cells.)
$c = 0$	ph = 13.0 mm.	ph = 16.0 mm.
5	13.5 „	17.7 „
10	14.0 „	18.8 „
15	18.? „	18.5 „
20	17.5? „	16.1 „
30	... „	16.? „

The zero points were 10.5 mm., 29.5 mm., and 38.5 mm., nearly, as before.

No experiments have as yet been made upon nickel or cobalt, the stress of the whole investigation having been thrown upon soft iron. The results of a long series of experiments similar to those quoted above, with slight modifications introduced by the method described below, and corrected as far as possible by the ordinary processes of differencing, are embodied in the large Table for Phase Retardation at the end of this paper. A set of curves of constant retardation are also given.

Although these experiments were conducted with the greatest exactness possible with the rough apparatus employed, they were open to criticism in that they were separated by long intervals of time, and possibly subject to wide variations of condition. To confirm their general results, and in a manner to leave no doubt that the retardation of phase really depends upon the speed of the commutator (the other conditions being constant), so as to gradually disappear and reappear with the diminution and increase, respectively, of its velocity, an electric motor was employed, whose revolving wheel was heavy enough to keep itself and the commutator in motion for a long time after the electricity had been cut off. In coming thus gradually to rest, all velocities between maximum and zero must have been passed through, thus affording the best possible opportunity for a close comparison.

Meantime, each swing of the galvanometer was noted (its period being made sufficiently great), as well as the exact time of stopping of the machine, obtained by a sharp signal from an assistant. The mean between two successive points of turning gave the average deflection of the galvanometer for the interval of time between them; and knowing the length of time of each swing (a constant), the rate of stopping of the machine (nearly constant), and the exact time of

its coming to rest relatively to the last recorded swing, it was easy to calculate the number of revolutions per second corresponding to a given deflection.

Taking different distances along the bar, as well as different inclinations of the two commutators, over a thousand determinations were made in the course of a few days. The result was in general a confirmation of the previous and more exact experiments.

In many cases an actual reversal of the current took place during the period of stopping of the machine; but of course when the inclination of the commutators was such that a reversal would indicate a retardation of more than the maximum found before, no such reversal took place.

These experiments serve, moreover, nicely to illustrate the fact upon which all thorough understanding of the above phenomena is based, namely, that magnetism requires a fixed time to reach a fixed value, and a greater time, proportionally, for greater distances from the magnetizing coil; so that, with even the moderate velocities employed (20 to 30 reversals per second), it has not time enough to reach its maximum even within the primary itself between two successive reversals; and for still shorter intervals of time reaches a proportionally less value.

This is proved by the fact that, when the commutators are adjusted to best advantage, the deflection of the galvanometer is independent of the speed of the commutator within certain limits, as 5 to 12 revolutions, or 10 to 24 reversals per second, after which it falls off a little; gradually diminishing more and more nearly in proportion to the increase in velocity, until, at 140 reversals, it is reduced to a quarter or a thirtieth of its maximum value, the diminution being most marked for distances moderately remote. For, if each wave had time to reach its maximum, the deflection, instead of diminishing, would increase in direct proportion to this velocity, as is indeed the case when the velocity is only a few turns per second. The actual results are contained in the Table of Galvanometer Deflections, which is appended.

It is evident that, by dividing the deflections contained in this table by the corresponding number of revolutions of the commutator, we obtain a series of numbers proportional to the actual height of each wave at the end of the short intervals of time during which the current acted. In the table below, the first column shows the duration of each current in terms of an arbitrary unit; the second column shows the corresponding height of the wave at a distance of two inches from the primary, expressed in percentages of that height

which it would finally have attained had the current not been cut off. The third column expresses similarly the percentage magnetization after different intervals of time at a distance of 14 inches along the bar. The fourth column is taken from a table in Jenkin's "Electricity and Magnetism," page 330, and expresses the percentage electrification at the end of a cable, also in terms of an arbitrary unit. The value of this unit for the French Atlantic cable is given as 0.196 seconds. The magnetic value of this unit for a distance of two inches over our bar is 0.010 seconds, and for fourteen inches 0.013 seconds nearly, or about one fifteenth of the electric value for the whole Atlantic cable. From this an idea can be formed of the greatness of magnetic, as compared with electric, resistance.

TABLE OF PERCENTAGE MAGNETIZATION AS DEPENDENT UPON THE TIME.

	$d = 2$	$d = 14$	For Electricity.
$t : a =$ 1.1	3.0	0.045	0.04
2.3	13.0	0.63	4.6
3.8	...	7.2	22.0
4.1	...	8.0	26.0
4.5	...	17.0	32.0
5.0	42.0	19.0	39.0
5.6	46.0	32.0	46.0
6.5	64.0	38.0	55.0
7.5	68.0	59.0	64.0
8.5	72.0	67.0	74.0
13.0	88.0	87.0	89.0

Of course it is not possible to expect close agreement of our results with those obtained for electricity. It is true that every law of magnetic induction may be said to correspond to one in electric conduction; and similarly we may expect that the laws of the establishment of magnetic induction (propagation of magnetism) will correspond to those of the establishment of the electric current. But we must remember that a bar magnetized at one end really corresponds not to a perfectly, but to an imperfectly, insulated cable, the laws for which I have not been able to find anywhere developed. The apparent agreement of the third and fourth columns of our table at the extremes, and to so large an extent elsewhere, is certainly remarkable whether accidental or not.

The table shows that the form of the magnetic wave at a moderate distance from the primary is similar to that of the electric wave at the end of a long cable, but that as we approach the primary the roundness disappears, so that a rapid series of waves would present more

or less sharpened crests and troughs. In no case, however (except for the lowest possible velocities), would the wave approximate to the rectangular form assigned to it by the theory of instantaneous action; nor, since its form differs at different distances, can it conform to any theory of instantaneous propagation.

The table also enables us to estimate the length of time that elapses before the wave can reach its mean height within the secondary when placed as near as possible (say two inches) to the centre of the primary. The time is about .058 seconds. This large retardation accounts for certain discrepancies found in earlier determinations of the zero points by the use of insufficiently low velocities.

The table shows, moreover, the actual velocity of propagation. We find, *e.g.*, that the time occupied by a point at fourteen inches in reaching fifty per cent of its maximum magnetization is about .035 seconds greater than for a point at two inches, giving a velocity of about twenty-eight and a half feet per second, which agrees as closely as could be expected with early results. For other percentages of magnetization we should of course obtain different results; but this particular result is of special significance in that it represents, in a measure, the rate of propagation of the body of the pulse. It may conveniently be termed the *principal velocity* of the magnetic wave.

The last method of measurement is likewise adapted to the use of a dynamometer (the phase-adjustment being a hinderance rather than a help); and is really an independent way of determining the velocity of propagation.

There is still another method of arriving at this velocity. If in the Table for Phase-Retardation, we subtract the phase for two inches from that for fourteen, and divide by the whole number of degrees described by the commutator in a second, we shall have the length of time occupied by the phase of mean height of the wave in travelling over the same twelve inches of the bar. The lowest possible velocity of revolution must be chosen in order to approximate to our former conditions.

Taking two revolutions per second, we find a difference of 24°, corresponding to one thirtieth of a second; whence the velocity is thirty feet per second, nearly as before.

The following is a series of velocities computed in the same way, each over six inches of the bar, beginning at 0, 2, 4, &c., inches: —

d.	Vy.	d.	Vy.	d.	Vy.
0	33	16	120	32	-45
2	26	18	180	34	-40
4	28	20	∞	36	-45
6	30	22	-180	38	-40
8	36	24	-120	40	-40
10	45	26	-90	42	-40
12	72	28	-60	44	-60
14	90	30	-51	46	-90
				..	∞

The minimum positive velocity has, of course, the greatest significance, being most free from the effects of direct action; and we may finally conclude that the "principal velocity" of propagation of the magnetic wave over a half-inch rod, unaffected by foreign influence, is in the neighborhood of twenty-five feet per second. The "principal velocity" of the electric pulse over the French Atlantic cable is about 13,700,000 feet per second.

Besides the experiments already alluded to, there was a preliminary series made with a Siemens armature machine, — the armature being converted into a powerful electro-magnet, and caused to revolve by means of a band and pulley-wheel. A series of magnetic waves was induced in the fixed horseshoe between whose poles the armature revolved; and, on screwing an iron rod firmly into one of its extremities, the waves spread more or less over the bar. A small induction coil was placed around the bar at a distance of sixteen inches, and connected with the galvanometer through a commutator which acted at the moment when the armature reached the position of greatest attraction. It is evident that, when the machine was set revolving very slowly, the impulses were all added together. It was found, however, as expected, that, as the velocity increased, the deflection of the galvanometer diminished, and was finally completely reversed, showing that a retardation of more than 90° of phase must have taken place. The least velocity which could cause a reversal was usually about fifteen revolutions per second, — although the number varied greatly at different times. It is interesting to observe that this velocity and distance correspond to a retardation of phase, in our table, of 108°.

It was thought that by this method the uncertainty of the time of magnetization of a primary coil would be got rid of, since the exact time of each impulse could be known; but the imperfection of the apparatus and discordance of the results caused this method to be abandoned.

The phenomena of the propagation of the magnetic wave in a bar

of soft iron are complicated by the fact that in addition to the pulse which reaches a given point at a given time, necessarily later than the time of its starting, there is another pulse, due to the direct action of the primary and adjacent parts; a pulse which at great distances becomes, as before stated, indefinitely large in comparison with the former. This will be evident on considering the nature of the two curves which represent the action of each pulse. The direct pulse decreases, of course, as the cube of the distance increases; while the logarithm of the pulse propagated through the bar, as determined by actual experiment, becomes regularly less in proportion to the distance. For a distance of fifty inches on our half-inch bar, whose coefficient of retained magnetism was 0.77 per linear inch, we have the direct pulse, by calculation, some three and three-fourths times greater than the indirect; so that, no matter what be its difference of phase, the effect cannot greatly exceed a retardation of 15°, and the point of maximum retardation, which was more than 40°, must lie somewhere on the bar. This was, as we have seen, the fact. It is easy to show, moreover, that this point cannot be nearer the primary than the point where the ratio of the indirect pulse to the direct (which it greatly exceeds) is at its maximum; and the latter is found by calculation to be at about 11.6 inches. The actual position of the point of maximum retardation of the phase lay always between these two limits.

As regards the phase of the primary itself, it is easily shown that since the increase of its magnetism determines an electromotive force opposed to that of the battery, which it cannot exceed, its rate of magnetization is determined for short intervals of time by that electromotive force; and that, in consequence, the magnetism keeps pace with the time.

In the differential equation

$$dm = \frac{\log^{-1}}{k}\left(\log E - \frac{Rt}{k^2}\right) dt;$$

put $t = 0$, and we have

$$m = \int \frac{E}{k}\, dt = \frac{E}{k}\, t,$$

the constant of integration being zero; this value of m being a special solution of the general integral,

$$m = \int_0^t \frac{\log^{-1}}{k}\left(\log E - \frac{Rt}{k^2}\right) dt = -\frac{Ek}{R}\left(\frac{1}{\log^{-1}\frac{Rt}{k^2}}\right) + \frac{Ek}{R},$$

where m = the magnetization, k = a constant depending upon the length, shape, etc., of the coil, R = the resistance of the primary circuit, and t = the duration of the current.

The phase of mean height will therefore be just half-way between that of maximum and minimum, or 90° later than it would have been were the rise of the magnetic wave instantaneous.

The phase is still more retarded in moderately distant portions of the bar, because the magnetization there depends, not merely upon the direct pulse, but also upon displacements already existing in the bar; but since these displacements agree in phase with the primary, and totally disappear with the return of the wave to its mean height, and since these displacements cannot exert a much greater influence than the primary itself (which causes them), the greatest possible retardation of phase due to these secondary effects cannot greatly exceed 45° or 50°, and may not reach that limit. We need not therefore be surprised that the total retardation is limited to 90° plus the above amount, or, in practice, to 130°.

I have left out all discussion of the oscillations of the induced current, since these will not, as a whole, interfere with the results. They are discussed in Gordon's "Electricity and Magnetism," vol. i. page 311 et seq.

I have already alluded to the experiments of Professor Blaserna upon a subject so closely allied to my own. The determination of the interval of time between the establishment of the primary current and the establishment of the induced current gives rise to a series of values of the rate of propagation of the pulse through air and other non-conductors, just as my own experiments have endeavored to do through iron. The results of the latter would indicate that the assumption apparently made by Professor Blaserna that the rate of propagation through air, etc., is constant, is at least open to criticism.

The obvious application of the results of the above investigation to the Gramme ring, the construction of electro-magnetic engines, and the whole subject of magneto-acoustics, I am obliged, for the present, to leave untouched; together with the exact mathematical solution of the problem of phase-retardation.

We have seen that magnetism, like electricity, can be said to have no proper velocity; that when one end of a conductor is magnetized or electrified, the other begins instantaneously to feel *an* influence, however slight; that the electric or magnetic pulse is not measurable for a short time, then rises very rapidly with the increase of time, then less and less rapidly, and finally becomes constant. Not only

qualitatively are the two sets of phenomena alike, but also, to a large extent, quantitatively; and there are reasons more than sufficient to account for their differences. We are led, therefore, to conclude that the laws of magnetic propagation are in all probability the same as, or very similar to, those of the establishment of the electric current; and consequently to the corresponding laws of heat.

SUMMARY.

I. The magnetic waves induced in a bar of soft iron by an alternating current differ in form, phase, and magnitude from the values assigned to them by any theory of instantaneous propagation.

II. They differ in form, inasmuch as the rise and fall of the magnetic wave is in no case abrupt; but it presents, even for a moderate number of reversals to the second, a perfectly rounded crest and trough, especially at considerable distances from the primary.

III. The magnitude of the waves is constant up to five or six reversals per second, after which it is much lessened; from twenty to thirty reversals, it diminishes nearly with the number of reversals (the deflection of the galvanometer remaining constant); from fifty to one hundred and forty it falls off more and more rapidly, especially for moderate distances along the bar, being there reduced so as to give about one thirtieth of its former maximum deflection.

IV. The phase within the primary is retarded more and more as the rapidity of alternation of the current increases, — to a maximum of 90°, and for a distance not too remote is invariably later than it is within the primary.

V. The apparent retardation of the phase depends upon the period (length) of the wave; the arbitrary distance from the primary; the material, dimensions, and special condition of the bar; and even, in extreme cases, upon the strength and distribution of the magnetic field.

VI. The retardation increases for a certain distance (about thirty diameters) to a maximum of from 127°, to 130°; or from 37° to 40° later than that of the primary; falling, again, gradually, at great distances, to 90°, that is to the original value within the primary itself.

VII. The apparent velocity of propagation, accordingly, increases

from a moderate value, in parts of the bar not too remote, to infinity, somewhere in its central portion, and then descends with change of sign, for a still further advance in distance, to a minimum, and finally becomes infinite again; so that, the most distant portions of the wave reaching their maximum height simultaneously with the nearest, the wave appears to flow backward over the most distant part of the bar.

VIII. The apparent velocity of propagation over the first fourteen diameters is from twenty-five feet per second, corresponding to four reversals per second, up to ninety feet, for twenty-three reversals, after which it rises nearly in proportion to the number of reversals, until, for one hundred and forty reversals, it measured about three hundred feet per second.

IX. The magnetic conductivity of the bar increased (in one case from two to three fold) under the continued influence of the alternating current sent by a high electromotive force, whose use in these determinations must generally be avoided.

X. The retardation of phase is less for small rods than for large ones, but in most respects the effects are similar for distances proportional to the relative diameters of the bars.

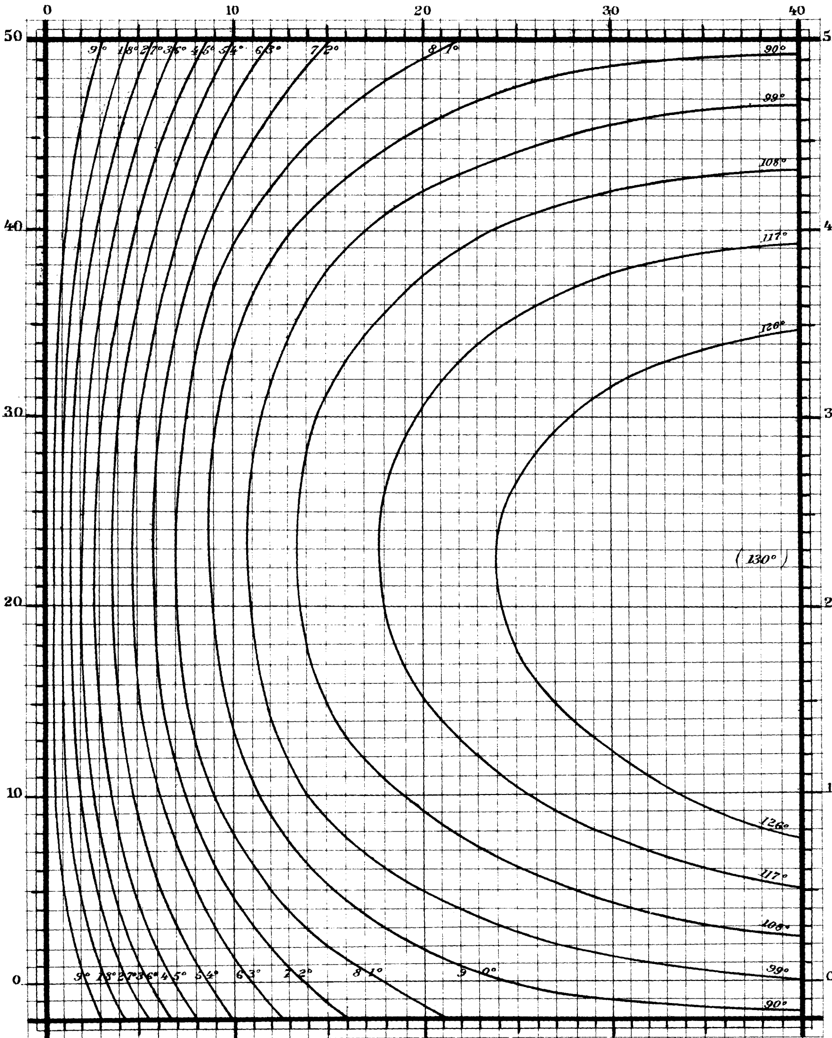
XI. The retardation is less for steel than for iron. With steel, a high electromotive force may be advantageously employed to overcome the tendency of the magnetic displacements to become permanent.

XII. The phase of the magnetic wave may be treated as the resultant of that of two sets of waves, one due to the direct (and instantaneous) action of the primary itself, and proportional, accordingly, to the time and to the inverse cube of the distance; the other depending upon the secondary effect of magnetism already induced in the bar, and hence proportional more or less to the square and higher powers of the time, and to the distance as the exponent of a constant factor, less than unity. In the same way that each law of magnetic induction has a parallel in the conduction of electricity, it is probable that the law of change in the one is similar to the law of change in the other.

TABLE FOR PHASE-RETARDATION.* (For mean height.)

v =	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	= v
d = 52	0	7	16	29	42	54	62	69	73	76	79	81	82	83	84	85	85	86	86	87	87	52
50	0	8	20	34	47	60	68	74	78	82	85	87	88	91	92	93	93	94	94	95	95	50
48	0	9	23	38	53	65	72	79	83	87	90	93	95	96	97	99	100	100	101	101	102	48
46	0	11	27	43	58	70	77	84	88	92	96	97	100	101	103	104	105	105	106	107	107	46
44	0	14	32	48	63	75	82	89	93	97	100	102	105	106	107	109	110	111	111	111	111	44
42	0	18	36	54	68	80	87	93	97	102	104	106	109	110	111	113	114	114	114	115	115	42
40	0	20	40	58	72	83	91	96	101	104	107	110	113	114	115	116	117	118	119	119	120	40
38	0	23	43	61	76	86	94	99	104	107	110	113	115	117	119	120	121	122	123	124	125	38
36	0	26	46	64	79	89	96	102	106	110	113	115	118	120	121	122	123	124	125	126	126	36
34	0	29	50	68	82	91	99	105	109	112	115	118	120	122	124	125	125	126	126	127	127	34
32	0	31	52	70	83	93	100	106	110	114	118	121	122	124	125	126	126	126	127	127	128	32
30	0	33	54	71	84	94	101	107	112	115	119	122	124	125	126	126	126	127	127	128	128	30
28	0	34	56	72	84	95	102	108	113	117	119	122	124	126	126	126	127	128	128	128	129	28
26	0	35	56	73	85	95	102	109	113	118	120	123	125	126	126	127	128	128	128	129	129	26
24	0	36	57	73	85	95	103	109	113	118	120	123	125	126	127	127	128	129	129	129	130	24
22	0	36	56	72	85	95	103	109	113	118	121	123	125	126	127	127	128	128	129	130	130	22
20	0	35	56	72	84	94	102	108	112	116	120	121	123	126	127	127	128	128	129	129	130	20
18	0	34	55	70	83	93	101	106	111	115	119	120	122	124	126	127	127	128	128	129	129	18
16	0	33	54	68	81	91	99	105	109	113	116	119	121	123	125	126	127	127	128	128	128	16
14	0	31	52	65	77	88	96	102	106	110	113	116	119	121	123	126	126	127	127	127	128	14
12	0	29	48	62	74	85	93	99	103	106	110	113	115	118	120	123	124	126	127	127	127	12
10	0	25	46	60	70	80	89	95	100	104	106	109	111	114	116	118	120	122	123	125	126	10
8	0	21	41	55	66	75	84	90	95	99	102	105	107	109	111	113	115	117	119	120	121	8
6	0	17	36	51	61	70	79	85	90	94	97	101	103	104	105	107	109	110	111	113	113	6
4	0	11	30	46	56	65	73	79	84	88	92	94	96	98	100	102	104	105	106	106	106	4
2	0	7	22	39	50	60	67	73	78	81	84	87	90	92	93	94	95	96	96	97	97	2
d = 0	0	6	16	30	44	54	61	67	72	76	79	81	82	83	84	85	85	86	86	86	86	0
v =	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	= v

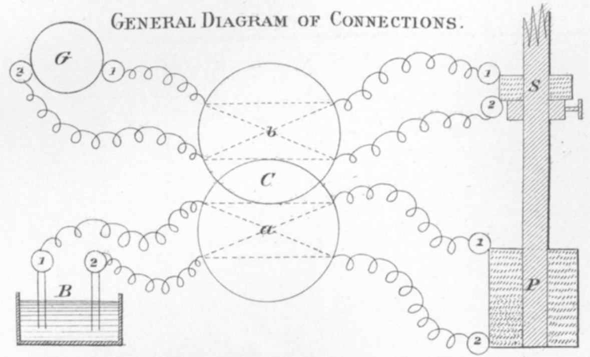
* See note under next table, also page 327 et seq.



CURVES OF CONSTANT PHASE RETARDATION.

See Table for Phase Retardation.

General Diagram of Connections.



B = the battery.
P = the primary.
S = the secondary.
G = the galvanometer.
C = the commutators.
(a) in primary circuit.
(b) " secondary "

r = the revolution-counter.
f = the fly-wheel.
w = the pulley-wheel.
t = the contact pieces.
s = the sliding piece.

Subscript numerals denote connections.

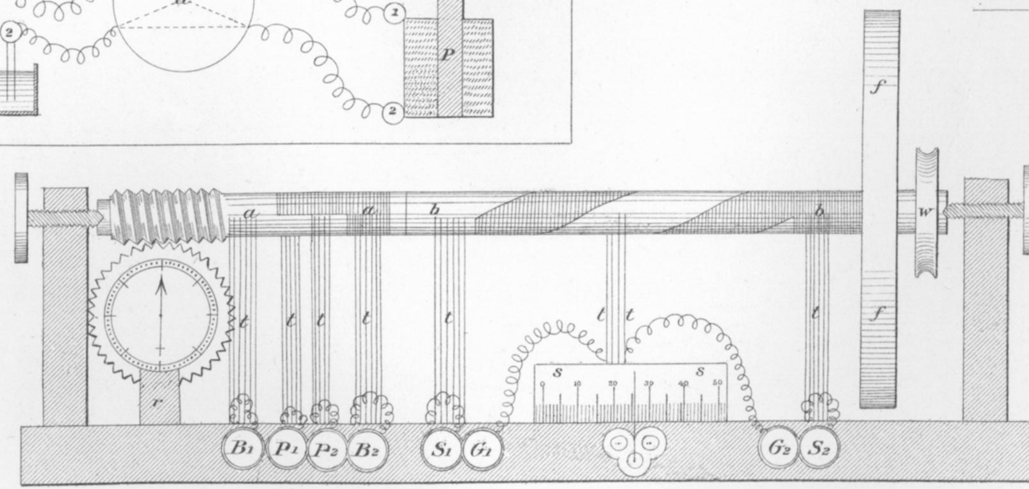


Diagram of the Double Commutator showing its Internal Connections.

TABLE OF GALVANOMETER DEFLECTIONS

	$d = 2$	5	9	14	22	32	47
$v = 1.6$	5	5	4	1	0.3	.020	.000
3.3	12	11	8	2	0.6	.038	.005
5.0	20	16	10	3	0.9	.057	.008
6.6	27	21	11	4	0.9	.072	.012
8.7	33	28	11	4	0.9	.090	.013
10.0	32	28	11	4	0.9	.081	.014
11.6	38	28	12	3	0.8	.081	.015
13.3	41	26	13	3	0.8	.081	.015
15.0	43	25	11	2	0.7	.081	.015
16.6	42	24	10	2	0.7	.081	.014
18.3	39	23	8	1	0.6	.079	.014
20.0	36	21	7	1	0.5	.079	.014
$(v = 0)$	1.37	0.69	0.24	0.06	0.01	.001	...
	$d = 2$	5	9	14	22	32	47

The phase retardation is measured in degrees, one whole phase (or period between two successive maxima or minima) being taken as 360°. The galvanometer deflections (see page 332 et seq.) are reduced to a common unit through the known ratio of various shunts, the greatest divisor being two hundred. The deflections due to a single closing of the circuit, always in the same direction, are placed in line with "$v = 0$." The divisions of the scale were about 5 mm. apart. d and v represent in both tables the distance in inches between the coils, and the velocity in revolutions per second. The galvanometer deflections are, of course, only approximate; but for small distances and low velocities, the probable error of the phase-retardation will not greatly exceed 5°. The curves of constant phase-retardation, at intervals of 9°, are represented, graphically, below; d and v being understood as vertical and horizontal coördinates.